A Network Model of Dairy Cow Replacement under Herd Constraints

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Abstract

A network model was developed to optimize dairy cattle replacement decisions under herd constraints. Herd constraints are limited resources, such as a limited supply of labor or heifers, which cause optimal replacement decisions for individual cows to be dependent of decisions for other cows in the herd. Consequently, optimal replacement decisions for individual cows under herd constraints may differ from optimal decisions when no herd constraints are present. Optimal replacement decisions need to be determined simultaneously for all cows in the herd. Previous approaches provided complex and approximate solutions, while exact solutions for realistic problems have not been developed. The approach presented here is to formulate the dairy cattle replacement problem as a network and solve it with a state-of-the-art network optimizer. The network model was developed in SAS 9.0 with the data step and was solved with SAS procedure Netflow. An example with a varying heifer supply showed that the model optimized individual culling decisions for cows in the herd, found the optimal herd replacement rate, and left some slots (stalls) open when heifer supply was limited. The formulation solves the dairy cattle replacement problem under herd constraints for individual cows at a realistic level, but computations may be time consuming.

Key words: Dairy Cattle, Replacement, Constraint Optimization, Network.

1 Introduction

To aid decision makers with dairy cattle replacement decisions, computer models have been developed that calculate the optimal time to replace a cow (e.g. van Arendonk, 1985; Kristensen, 1987; DeLorenzo et al., 1992; de Vries, 2004). These authors modeled the dairy cattle replacement problem as a Markov decision process and used value iteration (dynamic programming), or policy iteration to solve these sequential replacement decisions. These models calculated for each cow their retention pay-off (RPO, or future value). The RPO is the expected extra net return from keeping the cow until her optimal time of culling compared to culling her immediately, and can be used to rank the cows in the herd. All models, except the model of de Vries (2004), assumed that a culled cow was immediately replaced with a heifer.

Although some of these models were quite large and detailed, the optimal decisions for each cow were independent of decisions made for other cows in the herd. Thus, the only comparisons made were those between the current cow and the potential replacement heifer. The resources consumed by other cows in the herd were not considered for the optimization. However, herd constraints, such as a limited availability of heifers, labor or cash, may affect optimal decisions for individual cows. For example if the heifer supply is limited, the decision to replace some cows with the available heifers implies that for other cows no replacement heifers are available. Therefore, decisions for all individual cows need to be optimized simultaneously. This problem is known as the multi-component replacement problem in dairy cattle. This very large combinatorial decision problem has not been completely solved.

The objective of this study was to develop and describe a dairy cattle replacement model using a network formulation while maintaining the same level of detail present in the more detailed Markov decision
models, but with the possibility to add herd constraints. Results from the network model described here are illustrated with a varying supply of heifers.

2 Other approaches to herd constraint problems

Most realistic herd constraints imply that decisions made for some cows affect the optimal decisions for other cows in the herd. Although herd constraints can be modeled as an ordinary Markov decision process, the solution by value or policy iteration is computationally prohibitive (Kristensen, 1992).

Using the example of a limited heifer supply, Ben-Ari and Gal (1986) used a method called parameter iteration to obtain approximately optimal decisions for individual cows. Essentially, this is an iterative method which seeks to minimize a penalty function for violation of the herd constraint. This method was further developed by Kristensen (1992) with a much more realistic model than that used by Ben-Ari and Gal. However, the parameter iteration method is quite complex and does not provide exact solutions.

Jalvingh et al. (1994) used linear programming (LP) to determine the farm specific optimal herd calving pattern under seasonal conditions and herd constraints. Herd constraints considered were 1) milk quota, 2) heifer supply limited to calves born in the herd, 3) limits on variation in herd size during the year, 4) immediate replacement of culled cows. They first optimized the replacement decisions of a herd of cows that calved for the first time in the same calendar month, using the model of van Arendonk (1985). Herd constraints were not considered. Secondly, they imposed herd constraints to determine the optimal number of heifers calving in each month using LP. Thus, their replacement decisions were not optimal because they assumed immediate replacement and did not consider herd constraints for the calculation of the replacement decisions.

Houben (1995) first determined RPOs for individual cows assuming replacement decisions for cows were independent. Secondly, he used a genetic algorithm to adjust the replacement decisions (keep some cows longer, other cows shorter than the RPO suggested) to maximize profitability for a limited supply of heifers or a milk quota. However, individual replacement decisions may still not have been optimal.

Finally, Yates and Rehman (1998) developed the first known LP solution of a dairy cattle Markov decision process. Their herd constraint problem is from which cows to raise heifer calves that are going to be used as future replacements (assuming a surplus of heifer calves is available). The formulation appears correct, but their model is a simplified hypothetical example and not detailed enough for actual decision support on farms.

3 Network model

The dairy cattle replacement problem can be based on network diagrams that can be represented pictorially. A network consists of a collection of nodes joined by a collection of arcs (SAS Institute Inc., 2004). The arcs convey flow of commodities (cows) that are supplied at the supply nodes (the starting herd at time $t = 1$) and demanded at demand nodes in the network (the ending herd at $t = T$, for example one year in the future). Each arc has a cost per unit of flow and limits on flow capacity. Conservation of flow is maintained, meaning that the total flow in arcs directed towards a node, plus the supply at the node, minus the demand at the node equals the total flow in arcs directed away from the node (SAS Institute Inc., 2004).

In the network model described here, each cow is categorized by a combination of one of 9 milk production classes, 6 lactations, and 20 months in lactation. Furthermore, a cow is not pregnant or 1 to 9 months pregnant. A specific combination (category) is called a cow state. The model has monthly time steps (stages). Monthly seasonal differences in production and prices can be included.

Each node can be considered a cow state at a point in time while arcs carry the number of cows from one state (node) to another, subject to cost and constraints such as the minimum and maximum number of cows, the fraction involuntary culling, the fraction cows that get pregnant, and other constraints such as herd level constraints. The name of the node identifies the category. For example, the node 04B04020391 represents a cow state at the start of stage $t = 4$, 4th milk production class, 2nd lactation, 3rd month in
lactation, and 1st month of pregnancy (Fig. 1). In addition, notes representing empty slots are created at the beginning and end of a stage, for example node 04B00000000.

Arcs connect the nodes. For example, the arc 04B04020391_04E04020391 carries the cows in state (node) 04B04020391 through the stage to the end of the stage in node 04E04020391. The “cost” of this arc is the net return during this stage. Additional nodes carry the cows from the end of a stage to the start of the next stage at the cost of 0, for example, 04E04020391_05B04020492. Month of lactation and pregnancy states are incremented by one. Arcs to a node representing empty slots convey culling with a cost equal to the salvage value of those culled cows, for example the arc 04B04020391_04B00000000. Additional nodes and arcs are created, for example for cows that are in estrus and cows that are bred to distribute the proportion of cows that get bred and become pregnant, or remain non-pregnant.

Proportionality constraints may be applied to arcs to direct flow. For example, if the chance that a cow is observed in estrus is 40%, then a constraint is applied such that 40% of the total flow from that node is directed into an arc representing estrus detection and 60% is directed into the next node representing no detected estrus. Proportionality constraints are applied for the probability of estrus detection, conception, involuntary culling, and transition to other milk production classes. Furthermore, steady state constraints guarantee the same herd composition at the start of t = 1 as at the end of t = T. Additional constraints can be implemented such as on the arcs conveying heifers into the herd.

The network model was developed in SAS 9.0 with the data step. This step creates the framework of nodes and arcs. Secondly, economic and biological data such as costs and revenues, and the probability of estrus detection, conception, involuntary culling, and transition to other milk production classes are read from an external file and added to the constraints and arcs. The economic and biological data are obtained through user input in a user friendly spreadsheet (Excel). Thirdly, the flow through the network is optimized by Proc Netflow with the objective to maximize net return from t = 1 to t = T. This is equivalent to optimizing the net return of the herd over time, considering the constraints and applying optimal replacement decisions to individual cows. Due to the many proportionality constraints, the network problem is best solved by the Interior Point algorithm, which is called within Proc Netflow (the default simplex algorithm fails to solve the network problem). The Interior Point algorithm first converts the constrained network problem into an equivalent LP formulation, solves the problem, and then converts the results back into the network formulation.

The optimal solution consists of the maximum net return obtained by the network and the optimal flow through each arc. Herd statistics corresponding to the optimal solution can be calculate from the optimal flows, for example the number of empty slots, the number of cows in lactation 1 in each stage, the number of cows that are voluntary culled, and the number of heifers that enter the herd in each stage.

To illustrate the network model, data on lactation curves, prices, feed cost, labor cost, fixed cost, the risk of involuntary culling, estrus detection and conception rates were taken under conditions in Florida from the user input spreadsheet. Seasonal effects were not included. Net returns for each arc were calculated. Heifer supply was varied as a fraction of the number of available slots on the dairy farm. Two stages (T = 2) were used to guarantee steady state results.

4 Results

The network formulation resulted in 35,758 nodes, 79,947 arcs, and 43,124 side constraints. Proc Netflow typically used approximately 6 minutes to solve the problem, with the vast majority of the time spent on preprocessing the data. The equivalent LP problem, automatically set up by Proc Netflow, had 78,882 constraints and 90,749 variables. Typically around 40 iterations of the Interior Point algorithm were needed, depending on the constraints.

Herd summary statistics are shown in Table 1. When the supply of heifers was not constrained, net return / slot / year was $396.76 and 100% of the slots were filled with cows. Per year, 45.93% of slots received a replacement heifer. Because all slots were filled, this is equivalent to an annual cull rate of 45.93%. Forced entry of more or fewer heifers reduced net return. Not all slots were filled if the supply of heifers was limited. The results further indicated that the maximum net return / slot / year was not associated with the greatest milk yield / slot / day.
Figure 1. Part of the representation of the dairy cattle replacement network. Nodes are a categorization of a cow state or empty slot and are represented as vvBwwxyyzz (vv = stage, B {or E} = begin {or end} of stage, ww = milk production class, xx = lactation number, yy = month in milk, zz = pregnancy status (87 = not pregnant, ≥ 91 = pregnant)). Arrows indicate arcs that connect nodes.

Fig. 2 shows the percentage of slots filled by non-pregnant cows in their 2nd lactation, 6th milk production class, and up to 20 months in lactation. The optimal decision is to cull cows if no slots are filled. Otherwise the optimal decision is to keep the cows in those slots. Thus, the optimal decisions for individual cows are available in qualitative terms of keeping or culling the animals.

Table 1. Annual herd summary results from a variable supply of heifers.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Supply 1 24%</th>
<th>Supply 2 36%</th>
<th>No constraint</th>
<th>Supply 3 48%</th>
<th>Supply 4 60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net return ($/sot/year)</td>
<td>–73.16</td>
<td>255.07</td>
<td>396.76</td>
<td>395.14</td>
<td>355.11</td>
</tr>
<tr>
<td>Slots filled (%)</td>
<td>61.72</td>
<td>92.58</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Heifer supply (%)</td>
<td>24.00</td>
<td>36.00</td>
<td>45.93</td>
<td>48.00</td>
<td>60.00</td>
</tr>
<tr>
<td>Voluntary culling (%)</td>
<td>7.02</td>
<td>10.52</td>
<td>26.17</td>
<td>28.52</td>
<td>41.76</td>
</tr>
<tr>
<td>Involuntary culling (%)</td>
<td>16.98</td>
<td>25.48</td>
<td>19.76</td>
<td>19.48</td>
<td>18.24</td>
</tr>
<tr>
<td>Cull rate (%)</td>
<td>24.00</td>
<td>36.00</td>
<td>45.93</td>
<td>48.00</td>
<td>60.00</td>
</tr>
<tr>
<td>Cows in lactation 1 (%)</td>
<td>27.75</td>
<td>41.63</td>
<td>46.58</td>
<td>47.73</td>
<td>54.15</td>
</tr>
<tr>
<td>Cows in lactation 2 (%)</td>
<td>16.71</td>
<td>25.06</td>
<td>27.95</td>
<td>28.44</td>
<td>29.09</td>
</tr>
<tr>
<td>Cows pregnant (%)</td>
<td>27.84</td>
<td>41.76</td>
<td>49.43</td>
<td>49.52</td>
<td>48.15</td>
</tr>
<tr>
<td>Cows not pregnant (%)</td>
<td>72.16</td>
<td>58.24</td>
<td>50.57</td>
<td>50.48</td>
<td>51.85</td>
</tr>
<tr>
<td>Cows milking (%)</td>
<td>94.17</td>
<td>91.26</td>
<td>89.57</td>
<td>89.55</td>
<td>90.01</td>
</tr>
<tr>
<td>Cows dry (%)</td>
<td>5.83</td>
<td>8.74</td>
<td>10.43</td>
<td>10.45</td>
<td>9.99</td>
</tr>
<tr>
<td>Milk (kg/slot/day)</td>
<td>14.89</td>
<td>22.33</td>
<td>25.22</td>
<td>25.37</td>
<td>25.99</td>
</tr>
</tbody>
</table>

1 Supply is the percentage of slots receiving a replacement heifer per year.
2 Optimal supply determined by Proc Netflow.
5 Discussion and conclusions

It is possible to model a realistic dairy cattle replacement problem under herd constraints as a network in SAS and solve it with Proc Netflow. An advantage of the network formulation is that the problem can be drawn graphically, which helps model formulation and explanation.

The results show that the model is able to determine the optimal number of cows in the herd while making optimal replacement decisions for individual cows under the herd constraint of a limited supply of heifers. The results also show that individual replacement decisions depend on the herd constraint (Fig. 2). Other herd constraints can be implemented such as milk quota, labor supply, feed supply, milking parlor capacity, and more recently in the US a limited bovine somatotropin supply.

In addition to the optimization of replacement decisions, optimal breeding decisions for individual cows (when to start breeding cows and how long to continue breeding non-pregnant cows) can also be determined with the model. The effect of herd constraints on reproduction was explored by Risch and Wolf (2001) who found that the optimum calving interval under labor constraints significantly varied from the optimum when labor was not limited. However, it appears that they did not optimize individual cow decisions.

Another potential application is determination of the optimal length of the dry period, especially when parlor capacity is limited and dry cows can be housed and fed very cheaply. Furthermore, a logical and practical extension of the network formulation is to model a closed herd where the number of heifer calves born and raised in the herd determines the supply of heifers available for replacement. In addition, the tied heifer supply constraint might be relaxed by allowing heifer purchases and sales.

A disadvantage of the current model is that individual decisions are presented only as binary results, that is either keep or cull (and breed or do not breed). It is therefore not possible to rank cows based on the economic value of keeping the cow vs. the decision to cull her (the difference being the RPO). A possible method to obtain the RPO for an individual cow at t = 1 is to first determine the optimal decision (either keep or cull) and then solve the model again with the non-optimal decision for that cow forced into the model. The reduction in net return of the network is an approximation of the RPO for that cow. Note that the RPO determined under herd constraints is conditional on the constraint: if the constraint is lifted or altered, the RPO value for an individual cow may be changed.
In on-farm applications, one would use the existing number of cows in each cow state at the start of the first stage and define the desired herd distribution and number of cows after \( t = T \) stages. The model then is optimized for \( T \) stages. However, the model should be optimized again after one stage has passed and the then existing herd should be used as the starting herd.

The current network formulation can be solved in several minutes, but significant expansion of the network through inclusion of for example seasonality increases the time needed to solve the model from minutes to hours. Furthermore, the time to solve the network is dependent on the number of constraints. Most constraints regulate transition probabilities, such as transitions probabilities between milk production classes, the fraction detected estruses, and the fraction conceptions, and can therefore not be omitted.

The user friendly interface developed in Excel can be used to collect farm specific economic and biological data, as well as the herd constraints that are imposed. Currently, the user needs to have access to SAS Proc Netflow to solve the network model.

In conclusion, the network formulation of the dairy cattle replacement problem is a promising approach to optimize individual decisions for cows and empty slots under herd constraints.

6 References


